Code No: 133BD **R16**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B.Tech II Year I Semester Examinations, March - 2021

MATHEMATICS – IV

(Common to CE, EEE, ME, ECE, CSE, EIE, IT, MCT, ETM, MMT, AE, MIE, PTM, CEE, MSNT)

Time: 3 hours **Max. Marks: 75**

Answer any five questions

All questions carry equal marks

- - -

1.a) Find the analytic function whose real part is $e^x x \cos y \quad e^x y \sin y$.

b) Show that if u is harmonic and v is a harmonic conjugate of u, then u is a harmonic conjugate of v. [7+8]

2.a) Show that the function $x^2 + iy^3$ is not analytic anywhere. Reconcile this with the fact that the Cauchy–Riemann equations are satisfied at $x = 0, y = 0$.

b) Evaluate $\int_{0,1}^{2,5} 3x + y \; dx + \; 2y - x \; dy$ along:

i) the curve $y = x^2 + 1$,

ii) the straight line joining (0, 1) and (2, 5),

iii) the straight lines from (0, 1) to (0, 5). [7+8]

3.a) Integrate counterclockwise around the unit circle. Indicate whether Cauchy's integral theorem applies. Show the details.

$f \; z \; = z^3$

$f \; z \; = tan \; 1/4z$

b) Integrate by Cauchy's integral formula $z^2 / \; z^2 - 1$ counter clockwise around the circles $z + 1 \; = 1$ and $z + 1 - i \; = \pi/2$. [8+7]

4.a) Find the Taylor series with center z_0 and find its radius of convergence.

$1/ \; 1 - z \;, \quad z_0 = i$.

b) Find the Laurent series that converges for $0 < \; z - \; z_0 \; < R$ and determine the precise region of convergence. Show the details $\frac{e^z}{z-1 \;\; 2}$, $z_0 = 1$. [8+7]

5.a) Find all the singularities in the finite plane and the corresponding residues $\frac{sin\, 2z}{z^6}$

b) Evaluate counterclockwise $\int_c e^{1/z} dz$ C: the unit circle. [8+7]

6. Evaluate the following:

a) $\int_0^{2\pi} \frac{sin^2\theta}{5-4cos\,\theta} \, d\theta$

b) $\int_0^{\infty} \frac{dx}{1+x^2 \;\; 3}$ [8+7]

7.a) Is the given function even or odd or neither even nor odd? Find its Fourier series.
$f\ \ x\ \ = x^2,\ \ -1 < x < 1\ \ , p = 2$
And hence show that $1/6\ \pi^2 = 1 + 1/4 + 1/9 + 1/16$ ....

b) Find the cosine transform of $f_c\ \ w$ of
$f\ \ x\ \ = 1$ if $0 < x < 1$
$f\ \ x\ \ = -1$ if $1 < x < 2$
$f\ \ x\ \ = 0\ \ \ if\ \ \ x > 2$ [8+7]

8. Find the deflection $u\ \ x, t$ for the string of length L and $c^2 = 1$ when the initial velocity is zero and the initial deflection with small k (say, 0.01) is $k\ \ sin\pi x - \ 1/2\ \ sin 2\pi x$.
[15]

---ooOoo---